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Technical Report on the Research Activity
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P.13

The Title of the Research Project is:

"The Behavior of Elastic Anisotropic Laminated
Composite Flat Structures Subjected
to Deterministic and Random Loadings"

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(NASA-CR-192197) THE BEHAVIOR OF
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A) General Presentation of the Topics Analyzed Within This Project

Within this research project, the following topics have been studied:

- 1) Foundation of the refined theory of flat cross-ply laminated composite flat and curved panels as well as their static and dynamic response analysis.
- 2) Foundation of a geometrically-nonlinear shear-deformable theory of composite laminated flat panels including the effect of initial geometric imperfections and its application in the postbuckling analysis.
- 3) The study of the dynamic response of shear deformable elastic laminated composite panels to deterministic time-dependent external excitations as the sonic boom and explosive blast type-loadings.
- 4) The study of the dynamic response of shear deformable elastic laminated composite panels to random excitation as e.g. the one produced by a jet noise or by any time-dependent external excitation whose characteristics are expressed in a statistical sense.
- 5) The dynamic stability of fiber-reinforced composite flat panels whose materials (due to e.g. an ambient high temperature field) exhibit a time-dependent physical behavior.

B) Short Presentation of the Goals and Results
Obtained Within Each Topic

Topic (1): Within this topic the general goal was twofold:

- i) to develop a simple higher-order theory (HSDT) for the symmetric and antisymmetric anisotropic laminated composite flat and curved panels and
- ii) to apply a powerful solution technique based on the state space concept in order to evaluate in an exact manner their (static and dynamic) response behavior for a variety of boundary conditions.

The developed theory was aimed at preserving all the advantages of the first order transverse shear deformation theory (FSDT) both with regard to the number of involved unknowns and the order of the associated governing equations. However, in contrast to FSDT the developed theory is based on:

- i) an parabolic representation of transverse shear stress components across the panel thickness (thus eliminating the need for a shear correction factor); and
- ii) the elimination of the contradictory assumption involving simultaneous consideration of zero transverse normal strain and zero transverse normal stress.

The theory was developed by incorporating

- a) the anisotropy of the layer materials,
- b) the transverse shear deformation and transverse normal stress components,
- c) the rotatory inertia effects.

In addition, within this theory, the tangential conditions on the bounding external planes of the composite panel are exactly fulfilled.

The numerical results have been obtained for the case of flat rectangular or for curved panels whose projection on a plane is a rectangular one. While the edges $x_1 = \pm a/2$ have been considered arbitrary from the support conditions viewpoint (i.e. free-free (FF), free-simply supported (FS), free-clamped (FC), simply supported-simply supported (SS), simply supported-clamped (SC), clamped-clamped (CC)), the edges $x_2 = 0$, have been considered invariably simply supported (SS). The solution of the static and dynamic behavior of composite flat and curved panels for the above mentioned edge conditions was accomplished via a generalized Lévy-type solution considered in conjunction with the state space concept. The numerical results are presented in the published papers in the form of tables and graphs. They emphasize the effects of boundary conditions, in-plane orthotropy ratio, transverse normal stress, aspect ratio, thickness ratio, transverse shear deformation, on the static and dynamic response characteristics (as e.g. transversal deflection, eigenfrequencies, buckling loads, etc.).

Having in view that the transverse shear correction factor results as an intricate function of the geometrical and physico-mechanical lamination characteristics (as well as of the character of boundary conditions), one may infer the great role played by such higher order theories in the modelling of laminated plate theories, which in contrast to FSDT do not require the incorporation of shear correction factors. It should also be stressed that in addition to the developed theory, an exact computational tool allowing one to analyze numerically the

statical, as well as the response in buckling and free-vibration of flat and curved laminated composite panels for a variety of boundary conditions was developed. This constitutes a task which to our best knowledge was not attained so far. In addition, a thorough general analysis of the character of the statical and dynamical solution of flat composite panels was undertaken and several conclusions of practical importance have been outlined. In addition to these results, a study devoted exclusively to the formulation of the theory of antisymmetrically laminated composite flat panels was accomplished.

Topic (2): The goal pursued within this topic was twofold:

- a) to formulate a refined geometrically nonlinear theory of laminated shear deformable composite flat and curved panels and
- b) to analyze their postbuckling behavior by using this theory.

The refined nonlinear theory was developed by: 1) incorporating transverse-shear-deformation and transverse-normal-stress effects; 2) fulfilling the static conditions on the bounding planes of the panel; 3) incorporating higher-order effects and 4) incorporating the initial geometric imperfections. The necessity of introducing a transverse-shear correction factor used in first-order transverse-shear-deformation theory is removed. The theory is developed using the Lagrangian formulation. The three-dimensional strain-displacement equations were obtained by adopting the concept of small strains and moderate rotations. The equations associated with this refined theory are specialized for the case of cross-ply symmetrically-laminated composite

plates and, for this case, a formulation of the postbuckling behavior is presented.

Special attention was given to the case of the geometrically nonlinear theory of composite plates whose layers are made of transversely-isotropic materials. For this case it was shown that the associated governing equations may be recast in the form of two uncoupled sets of equations: 1) a set of equations defining the interior solution; and 2) an equation defining the boundary layer effect. The boundary layer solution decays rapidly when proceeding from the edges to the interior of the panel. As it was shown earlier in Reference 1[†], the boundary layer solution has negligible effect when dealing with problems involving global behavior, such as buckling, postbuckling, free vibration and flutter.

The postbuckling behavior in bi-axial compression of such laminated composite plates was investigated in detail by dealing both with the full system of governing equations and with the equations governing the interior solution only. The results are plotted in the form of graphs. They indicate the importance of including transverse shear deformation; transverse normal stress effect as well as the geometrical parameters of the composite panel. Finally, the results obtained by the use of the higher-order shear-deformation theory are compared with the ones resulting from the first-order transverse-shear-deformation theory and the classical Kirchhoff theory counterparts. A number of conclusions concerning the range of applicability of these theories are presented.

[†]Librescu, L., *Elastostatics and Kinetics of Anisotropic and Heterogeneous Shell-Type Structures*, Noordhoff Int. Publ., The Netherlands, 1975.

In addition several new results have been obtained in connection with the effects played by the initial geometrical imperfections on the postbuckling behavior in compression of flat composite panels. In this sense the imperfection-sensitivity on the postbuckling behavior was analyzed and pertinent conclusions were outlined. The results obtained within the framework of this topic reveal that: i) for the composite constructions whose layers exhibit either a low transverse shear modulus or relative thick cross sections, the effects of transverse shear are essential not only as concerns the prediction of the initial buckling but also of the postbuckling behavior and ii) the classical (Kirchhoffean) theory provides unreliable results both as concerns the bifurcation instability and the postbuckling behavior.

Topic (3): The response of elastic structures to time-dependent pulses, such as sonic boom and blast loadings constitutes a subject which is currently of much interest in the design of aeronautical and space vehicles. Its study was done in the past for metallic thin structural members. With the advent of the new composite material structures and their increased use in the aerospace industry, there is a need to reconsider the problem of the structural response.

Within our analysis the far field overpressures produced by an aircraft flying supersonically in the earth's atmosphere (referred to as the sonic-boom pressure pulse) or by an supersonic projectile, rocket or missile (referred to as the sonic-boom pressure pulse), as well as the one resulting from an explosive blast were used to predict the panel response. In the case of the sonic-boom, the time-history of the blast was described as an N-shaped pulse whose negative phase duration was included as a variable in the analysis. As concerns the explosive

blast, its time-history was approximated both as an exponentially decaying pressure pulse and, in an approximated way, by a triangular pulse. The cases of undamped and viscously damped oscillations of composite shear deformable panels exposed to such time-dependent excitations were analyzed.

The results obtained within the higher-order plate theory developed by L. Librescu (HSDT) were compared with their first order transverse shear deformation (FSDT) and classical (CLT) counterparts and a number of conclusions of practical importance have been outlined. In connection with the solution procedure which was used, this was based on the integral-transform technique. This has allowed one to express the response solution in a general form, in the sense that it was not restricted to a specific type of pulse loading, but to an arbitrary one, of given time-history.

Topic (4): The structural elements of the aerospace systems have to be designed as to withstand hostile environmental conditions which in many cases have a random character. In such cases it is required to obtain the response characteristics expressed in a statistical sense, in terms of the corresponding parameters of the external excitation (expressed also in the statistical sense).

Within the present study the case of the pressure field in a turbulent boundary layer; the case of a point load random in time (characterized by a constant spectral density) as well as the case of the pressure loads having a non-stationary random character (occurring e.g. during the launch phase of missile flight or by a blast) have been used in order to study the dynamic response of composite flat structures. The goal of this research was twofold: i) to determine the

response, expressed in statistical terms, to an arbitrary time-dependent random excitation and ii) to compare the results obtained within the framework of various theories, including the classical (Kirchhoffean) one. The numerical results obtained for a number of cases allow one to conclude that for non-thin composite plates (and shells) or for the ones which exhibit a low rigidity in transverse shear, it could be a large overshoot in excess to the stationary response calculated by the classical plate theory. These results could play a great role in a rational design of structures exposed to severe environmental conditions as well as in a reliability analysis of composite plates (and shells) subjected to random excitations.

Topic (5): Within this topic, a study of the dynamic stability of unidirectional fiber-reinforced composite viscoelastic plates subjected to compressive edge loads was performed.

The integro-differential equations governing the stability problem were obtained by using, in conjunction with a Boltzmann hereditary constitutive law for a 3-D viscoelastic medium, a higher-order shear deformation theory of orthotropic plates. Such a theory developed by Librescu incorporates transverse shear deformation, transverse normal stress, rotatory inertia effects and fulfills the conditions on the bounding planes of the panel. The solution of the stability problem as considered within this study concerns the determination of the critical in-plane edge loads yielding the asymptotic instability. Numerical applications, based on material properties derived within the framework of Aboudi's micromechanical model are presented and pertinent conclusions concerning the nature of the loss of stability (i.e., by flutter or divergence) and the influence of various parameters are outlined.

In addition, the dynamic stability analysis was performed for the case of viscoelastic plates exhibiting transversely-isotropic properties, the plane of isotropy being parallel to the mid-plane of the plate. Such transversely-isotropic materials are used in the thermal protection of high-performance vehicles. Due to the high temperature gradients experienced by these structures, their materials exhibit viscoelastic properties. The results obtained for the case of shear deformable viscoelastic plates reveal a diminishing of the load carrying capacity of the plate, in the sense that the bifurcation instability diminishes as compared both with the classical (Kirchhoffean) and the elastic models. In addition to this case two other ones of practical importance have been studied: a) the case of creep instability of orthotropic fibre-reinforced composite plates and b) of uniaxial fibre-reinforced composite plates.

C) Papers Published (or Under Print) and Supported by the Grant

Within Topic (1)

1. L. Librescu and A. A. Khdeir. "Analysis of Symmetric Cross-Ply Laminated Elastic Plates Using a Higher-Order Theory," Part I. State of Stress and Displacement in Composite Structures (An International Journal) 79, 1988, pp. 189-213.
2. A. A. Khdeir and L. Librescu. "Analysis of Symmetric Cross-Ply Laminated Elastic Plates Using a Higher-Order Theory," Part II. Buckling and Free Vibration, Composite Structures (An International Journal) 9, 1988, pp. 259-277.
3. L. Librescu, A. A. Khdeir and D. Frederick. "A Shear Deformable Theory of Laminated Composite Shallow Shell-Type Panels and Their Response Analysis," Part I. Free Vibration & Buckling, Acta Mechanica, 76, 1-33, 1989
4. A. A. Khdeir, L. Librescu and D. Frederick. "A Shear Deformable Theory of Laminated Composite Shallow Shell-Type Panels and Their Response Analysis," Part II. Static Response, Acta Mechanica, 77, 1-12, 1989.

5. L. Librescu and J. N. Reddy. "A Few Remarks Concerning Several Refined Theories of Anisotropic Laminated Plates," International Journal of Engineering Science, Vol. 27, 5, pp. 515-527, 1989.
6. L. Librescu, "Formulation of an Elastodynamic Theory of Laminated Shear-Deformable Flat Panels," Accepted for publication in Journal of Sound and Vibration (1990).

Within Topic (2)

7. L. Librescu and M. Stein. "A Geometrically Nonlinear Theory of Transversely-Isotropic Laminated Composite Plates and Its Use in the Postbuckling Analysis," Thin Walled Structures (An International Journal), (Special Issue Dedicated to Aeronautical Structures), 1990, (to appear). This paper will appear also in a book.
8. L. Librescu and M. Stein, "Postbuckling Analysis of Shear Deformable Composite Flat Panels Taking Into Account Geometrical Imperfections," AIAA-90-0967-CP, AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference, Part 1, pp. 829-902, Long Beach, CA, 1990.

Within Topic (3)

9. L. Librescu and A. Nosier. "Response of Shear Deformable Elastic Laminated Composite Panels to Sonic-Boom and Explosive Blast Loadings," AIAA Journal, Vol. 28, No. 2, February 1990, pp. 345-352.
10. A. Nosier, L. Librescu and D. Frederick. "The Effects of Time-Dependent Excitation on the Oscillatory Motion of Viscously Damped Laminated Composite Flat Panels," in "Advances in the Theory of Plates and Shells," Eds. G. Z. Voyiadjis and D. Karamanlidis, Elsevier Science Publ., Amsterdam, The Netherlands, 1990, pp. 249-268.
11. L. Librescu and A. Nosier, "Dynamic Response of Anisotropic Composite Panels to Time-Dependent External Excitations," Paper to appear in the Proceedings of the International Council of Aeronautical Science Conference (ICHS), August 1990, Stockholm, Sweden.

Within Topic (4)

12. G. Cederbaum, L. Librescu and I. Elishakoff. "Several Remarks on the Dynamics of Higher-Order Laminated Plate Theory and its Application in Random Vibration Response," International Journal of Solids and Structures, Vol. 25, pp. 515-526, 1989.
13. I. Elishakoff, G. Cederbaum and L. Librescu. "Response of Moderately Thick Laminated Cross-Ply Composite Shells Subjected to Random Excitation," AIAA Journal, Vol. 27, No. 7, 1989, pp. 975-981.

14. G. Cederbaum, L. Librescu and I. Elishakoff. "Random Vibrations of Laminated Plates Modelled within the First-Order Shear Deformation Theory," Composite Structures, 12, 1989, pp. 97-111.
15. G. Cederbaum, L. Librescu and I. Elishakoff. "Response of Laminated Plates to Nonstationary Random Excitation," Journal of Structural Safety, 6, 1989, pp. 99-113.
16. I. Elishakoff, L. Librescu and G. Cederbaum, "Stationary and Nonstationary Random Vibrations of Laminated Composite Plates via a Higher Order Theory," AIAA-90-0991CP, AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference, Part 3, pp. 1857-1869, Long Beach, California, 1990.

Within Topic (5)

17. Chandiramani, N. K. and Librescu, L. "Stability of Uniaxially Fibre Reinforced Composite Viscoelastic Panels," Applied Mechanics Review, Vol. 42, No. 11, Part 2, Nov. 1989, pp. 39-47.
18. L. Librescu and N. K. Chandiramani. "Recent Results Concerning the Stability of Viscoelastic Shear Deformable Plates Under Compressive Edge Loading," to appear in Solid Mechanics Archives (issue dedicated to H. Leipholz).

D. Papers Supported by this Grant and Presented
at National and International Conferences

Several papers elaborated within the framework of this Grant have been presented at international and national conferences. It is the case with the papers:

L. Librescu and M. Stein. "Nonlinear Theory and Postbuckling Behavior of Shear Deformable Symmetrically Laminated Composite Panels," 16th ICAS Congress of Aeronautical Sciences, Jerusalem, Israel, August 1988, ICAS-88-5.2.4 ICAS Proceedings, Vol. 1, pp. 349-160, 1988.

L. Librescu, A. A. Khdeir and D. Frederick. "Free Vibration and Buckling of Cross-Ply Laminated Shear Deformable Shallow Shell-Type Panels," Proceedings of the Third International Conference on "Recent Advances in Structural Dynamics," 18-22 July 1988, University of Southampton England, pp. 229-239, Eds. M. Petit, H. F. Wolfe and C. Mei.

L. Librescu and M. Stein. "The Use of Higher Order Theory in the Postbuckling Analysis of Shear-Deformable Symmetrically-Laminated Composite Flat Panels," Fifth Annual Review, Virginia Tech. Center for Composite Materials & Structures, April 4-6, 1988, Blacksburg, VA.

I. Elishakoff, G. Cederbaum and L. Librescu. "Random Vibrations of Moderately Thick Laminated Cylindrical Shells Excited by a Ring Loading," XVIIth International Congress of Theoretical and Applied Mechanics, August 21-27, 1988, Grenoble (France).

L. Librescu and M. Stein. "Postbuckling of Shear Deformable Composite Panels Taking into Account Geometrical Imperfections," Sixth Annual Review, Virginia Tech Center for Composite Materials and Structures, 9-11 April, 1989.